# THE IMPLICATIONS OF THE FUNDAMENTAL FORMULAS FOR FREQUENCY SELECTION IN ULTRASONIC PLASTICS WELDING

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## Abstract

This paper examines the fundamental formulas relating to heating, attenuation, waveguide size, mass and momentum, and applies them to the process of ultrasonic plastics welding. The question of the relative merits of higher- or lower-frequency ultrasonic equipment for various configurations and materials are discussed in terms of the physical properties of those materials, and the desired outcome. This is especially relevant to the application engineer's primary task, namely to determine if the appropriate amount of amplitude will be available with equipment and tooling of a given frequency.

## Background

Much of the work of both applications and tooling engineering professionals in the ultrasonic plastics welding industry is actually spent on one task: Assuring that there will be sufficient amplitude at the joint area to produce the desired effect without doing damage to other areas of the assembly. It is this balancing act that lies as the backdrop for each decision made in the development of an applications feasibility report and in the development of tooling that will be both functional and durable. The horn designer must pay particular attention to designing a tool that will produce the desired amplitude without fracturing, while the application engineer is constantly striking a balance between enough amplitude but not too much.

Adding to the difficulty of the modern application engineer's task is the pace with which new thermoplastic materials are added to the part designer's ammunition box. The sheer number of different basic resins, grades, and additive combinations has made applications work a combination of good engineering practice and street smarts about how various resins will react to the ultrasonics assembly process. In addition, the dissemination of solid modeling CAD technology has opened up to designers a whole new universe of part geometries that formerly were not widely used because of the impracticality of physical modeling using patterns and the like to produce complex mold shapes.

Further complicating the landscape is the proliferation of available frequencies in ultrasonic welding equipment. Whereas in 1980, only two frequencies were commonly in use in North America, two decades later at least one manufacturer of ultrasonic equipment has expanded their offering to five different frequencies ranging from 15KHz to 70KHz.

All of these developments mean that a solid understanding of the fundamental formulas and their implications is critical in order to make solid applications engineering decisions. The additional frequencies are a blessing in terms of being able to assign the right tool to the job, but along with this flexibility comes the need to understand how some basic acoustic and mechanical principles bear on the frequency selection decision.

## Caveats

To aid readability, not all of the steps in the algebra have been shown. Also, in most cases it is assumed that the work pieces are isotropic (consistent in physical properties in all directions) and boundless, and that waves and physical objects always behave as they should. If the reader is interested in the math, there is enough to go from, but if the reader is not, the formulas can be skipped without much loss of understanding.

There are five fundamental areas addressed in this paper where frequency and amplitude have effect on ultrasonic assembly applications. That there are five addressed does not mean that only five exist. This paper does not claim to be exhaustive. At the same time, effort has been made to cover as much of the most important material as possible.

### 1. Heating Rate

The fundamental formula regarding heating of materials via ultrasound[1] is:

$$Q_{avg}=\omega\varepsilon_o^2E''/2$$

Where: $Q_{avg}$ is the heating rate
$\omega$ is the frequency
$\varepsilon_o$ is the applied strain (proportional to the amplitude)
E" is the complex loss modulus of the material

The formula for heating materials ultrasonically can be factored to:

$$Q_{avg}=\omega\varepsilon_o\varepsilon_o\ E''(0.5)$$

In examining the effect of changing frequency, it is critical to remember that the amplitude is generally inversely proportional to the frequency, i.e., amplitude at 20KHz is usually twice what it is at 40KHz. Examining the above formula, algebra yields that for any given application, since the material is the same, the heating rate is affected by amplitude change and frequency change according to the following formula:

$$\Delta Q_{avg}\propto(\Delta\omega)(\Delta\varepsilon_o)(\Delta\varepsilon_o)$$

Again, if the frequency is doubled, amplitude will typically be halved, so in this particular case the heating rate will behave according the to the following formula:

$$\Delta Q_{avg}\propto 2\omega(\varepsilon_o/2)(\varepsilon_o/2)$$

Since the multiplication by two in the first term cancels the division by two in the second term, it can be seen that in this case,
the general rule is:

$$\Delta Q_{avg}\propto\Delta\varepsilon_o$$

This implies that the heating rate will vary directly with the change in amplitude, and therefore inversely proportional to change in frequency. We reason then, that using most generally available ultrasonic equipment, switching to a lower frequency will allow welds to be made more quickly.

### 2. Depth of Field

Some articles declare that the lower the frequency of the welder, the lower the attenuation through the material (loss of amplitude as distance increases)[2]. This is not true in the strictest sense. Benatar, Yarin, and Rittel's work with Kolsky bar testing has so far failed to show any link between attenuation and frequency for any frequency above approximately 15KHz.[3] In fact, it appears that the loss modulus might actually increase below 15KHz. Attenuation appears to be lower with lower frequency equipment because of the greater depth of field available with lower frequency equipment. There are two factors to consider when looking at the question of depth of field.

### 2a. Attenuation Effect

When discussing heating rate of a material, we generally do not address the thermal conductivity of the material. In the real world, however, what one seeks to do is to elevate the temperature of a localized area in a material. It can be readily reasoned that if the heating rate of the material is less than the rate at which the material can dissipate the heat generated, melt temperature will never be attained. So from a practical standpoint, it can be supposed that there is a threshold of amplitude for each material.

The formula for sound pressure[4] looks like this:

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[1] Handbook of Plastics Joining, Plastics Design Library, Div. of William Andrew Inc., New York, 1997, p.39

[2] Ibid, p.54

[3] Benatar, Avraham; Yarin, Alexander; and Rittel, Daniel, *Experimental Measurement of the Dynamic Moduli of Polymers Using an Instrumented Kolsky Bar*, The Ohio State University, Columbus, Ohio, and Technion – Israel Istitute of Technology, Haifa, Israel, Reference Number 3103, 2000

[4] *NDTnet*, downloaded August 24, 1999, www.ndt.net/article/ut_az/append_a/append_a.htm

$$p=\rho c\omega\varepsilon$$

Where: $p$ is the sound pressure
$\rho$ is the material density
$c$ is the speed of sound in the material
$\omega$ is the frequency
$\varepsilon$ is the amplitude

The formula for attenuation[5] looks like this:

$$p/p_o=e^{-\alpha d}$$

Where: $p$ is the sound pressure at the end of length $d$
$p_o$ is the sound pressure at the start of length $d$
$\alpha$ is the coefficient of attenuation for the material (normally units of attenuation are in decibels and the coefficient of attenuation is in decibels per unit length)
($d$ was already defined as the length)

One can readily see that the sound pressure would drop precipitously with small amounts of increase in distance in relatively lossy materials. But what characteristic of the sound wave actually changes with the distance?

Reviewing the four factors that constitute sound pressure, for any given application, density, the speed of sound, and (for purposes of this discussion) frequency do not change in an isotropic material, therefore the only change to consider is amplitude. Given that one must attain a certain amount of amplitude at the joint to attain a heating rate that exceeds the ability of the material to dissipate the heat created, one could calculate a boundary within the material (see Fig. 1). The side of the boundary closer to the source of sound pressure would be the zone in which joining can occur, and the more distant side of the boundary would be the zone in which there is insufficient amplitude for joining. It is easy to reason that that boundary is closer to the source for a lower amplitude source (see Fig. 2) than for a higher amplitude source (see Fig. 3).

Working the algebra for the formulas above and eliminating the constants, we find that:

$$D_f \propto \log_e \varepsilon$$

In English, at any given frequency, depth of field varies with the natural logarithm of the amplitude (see Fig. 4). This actually predicts a phenomenon that exhibits a law of diminishing returns, in other words, amplitude increases will increase depth of field, but the more amplitude one has, the less increase in depth of field is available.

Since in the sound pressure equation, frequency and amplitude are equal in their effect as factors, if we do change amplitude in inverse proportion to frequency, the greater depth of field must be related to some other difference in the way waves interact with solids.

**2b. Wavelength Effect**

For any given material, the lower the frequency, the longer the wavelength. The relevant formula[6] is:

$$\lambda=c/\omega$$

Where: $\lambda$ is the wavelength
$c$ is the speed of sound through the material
$\omega$ is the frequency

It can be seen that the wavelength is dependent on the stiffness (speed of sound) of the material. Since the frequency of a given ultrasonic system is for all intents and purposes fixed, the wave induced in a particular part will have a wavelength based on the frequency and stiffness. In common horn materials, the half wavelength for 20KHz is typically around 130mm. For 40KHz it is around 65mm, and for 15KHz it is typically around 175mm. Only a few of the thermoplastic materials are as stiff as the metals used to manufacture horns, however. Generally, the materials we join are considerably less stiff than the horn materials, and so, the wavelengths will be much smaller.

It may be proper to presume that a material will generally operate in a direct stiffness mode if it is less than a quarter wavelength thick (see Fig. 5). In other words, if the first nodal point would lie beyond the joint based on the speed of sound in the material, then the material will operate in a straight stiffness mode (it will vibrate as a unit

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[5] Ibid

[6] Ibid

and not go into waveform resonance). If, however, the first nodal point lies at or beyond the joint, the material will probably operate in a resonant mode, forming nodes and antinodes as ultrasonic tooling does (see Fig. 6).

The result of all of this is that the best ultrasonic joining is available in the first one eighth wavelength, which is often alarmingly close to the horn contact, especially for low stiffness materials.

For example, if a material is only one-eighth the stiffness of aluminum, the half wavelength at 20KHz will be only approximately 17mm from the horn contact, making the eighth wave distance only 4mm! For 40KHz, the one eighth wavelength drops to approximately 2mm. However, if the same application is moved to 15KHz, the one eighth wavelength goes up to approximately 5.5mm. This small gain may be enough to make an unfeasible application feasible.

## 4. Spreading Losses

The smaller a waveguide, the lower the spreading losses will be. The more energy that can be passed as a near-planar shock wave from transducer to horn face, the more efficient the transmission of energy will be.

The relative size of the waveguide is measured in relation to wavelength[7]. The longer the wavelength, the smaller a particular waveguide will appear to the wave. A 75mm diameter rod would be an extremely lossy waveguide for 40KHz, while a 75mm diameter rod would be an excellent waveguide for 15KHz. A common rule of thumb among horn designers is to avoid any single waveguide form (simple horn or cell of a slotted horn) that has any transverse dimension greater than about 1/3 wavelength.

This rule allows larger tooling to be built for lower frequency equipment (as a result of the longer wavelength), and allows a horn with the same transverse dimensions to be of a less complex design and therefore run better at a lower frequency (see Fig. 7).

When considering longevity of the tool, a simpler horn can always be driven at higher amplitude than a more complex horn. In addition, the ability to possibly use larger dimensions in the input quarter wave of the horn because of reduced spreading losses can introduce additional amplitude gain to the tooling. This can allow for an increase in amplitude that exceeds the proportional drop in frequency and solves the question of why attenuation often appears to be less at lower frequencies. The horn that is used with the lower frequency is most often simply a better waveguide.

This same principle allows for larger tools to be made that have acceptable performance from an applications point of view (i.e. delivery of sufficient amplitude). This means that some applications that are unfeasible at higher frequencies will be feasible at lower frequencies.

The longer wavelength also allows for a larger transducer to be built at a lower frequency. The more piezoelectric material that can reasonably be built into a transducer, the higher the power level that can be safely and reliably attained.

## 5. Mass, Momentum, and Clamp Force

Larger tooling has higher mass, thus more momentum when it is ringing, and can impart higher power levels to the workpiece. This is a reason for turning to lower frequency equipment for large or difficult applications.

To greatly simplify, it is widely understood that linear momentum is equal to the product of mass and velocity. In other words, if the same velocity can be achieved, a larger hammer does more work. In effect, the larger, lower-frequency horn is that larger hammer, and it therefore does more work, which is observed as a higher power level and faster welding.

Larger tooling and higher power levels allow for, and might even demand, construction of larger equipment capable of higher clamp forces. These higher clamp forces exert their own effect on accelerating the ultrasonic assembly process, working as a simple factor in the heating equation. Likewise, smaller and lighter built systems (with correspondingly higher frequencies) will have more responsive touch and will allow for more precise control of the assembly process.

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[7] B.A. Auld, Acoustic Fields and Waves in Solids, Volume II, John Wiley and Sons, New York, 1973, pp.64-65

## Observations

One would therefore expect a lower frequency system to weld a particular part faster, with higher clamp force and a higher power draw, than a higher frequency system. So why bother to even consider higher frequency equipment and tooling?

The answer is that the lower frequency equipment impacts workpieces more profoundly than higher frequency equipment, which is a good thing if the workpieces are large and difficult to pass sound through. If, however, the workpieces are small, delicate, and made of relatively stiff materials, the high-power, high-amplitude, "get-a-bigger-hammer" approach of lower frequency equipment can easily result in undesireable part damage such as fracturing and marking. Also, the faster welding at lower frequencies, coupled with the generally more massive equipment makes the process quite unwieldy to control when working with smaller and more intricate structures.

## Conclusions

For any given application, switching from higher frequency equipment to lower frequency equipment will usually result in:

1. More available amplitude
2. Faster welding
3. Greater available depth of field
4. Either larger or simpler tooling designs
5. More clamp force available
6. Increased collateral damage

For any given application, switching from lower frequency equipment to higher frequency equipment will usually result in:

1. Less available amplitude
2. Slower, more controllable welding
3. Smaller depth of field
4. Smaller, lighter, possibly more complex tooling
5. Smaller, lighter mechanical system
6. Reduced collateral damage

## Final Caveat

This paper has been an attempt to discuss some of the most important aspects of the effects of frequency and amplitude on ultrasonic assembly. It was not intended to be exhaustive, and may not address all of the factors to consider when faced with a frequency choice in an application. Until and if ultrasonic assembly is perfected as a science, experimentation will be the ultimate proof of which frequency is right for a particular application.

## Acknowledgements

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Any failures of logic, unfounded conclusions, or outright misinformation are the responsibility of the author alone.

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FIGURE 1. DEPTH OF FIELD CONCEPT

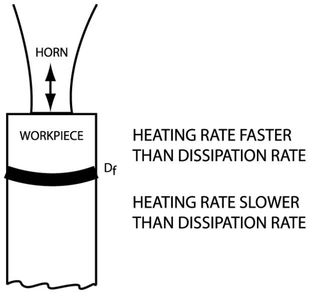


FIGURE 2. DEPTH OF FIELD NOT GREAT ENOUGH FOR APPLICATION SUCCESS

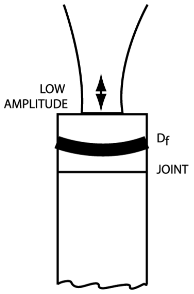


FIGURE 3. DEPTH OF FIELD GREAT ENOUGH FOR APPLICATION SUCCESS

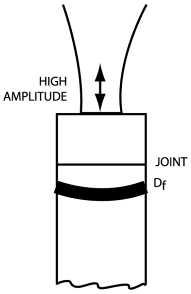


FIGURE 4. PURE ATTENUATION EFFECT

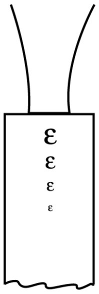


FIGURE 5. WAVELENGTH EFFECT - PURE STIFFNESS RESPONSE

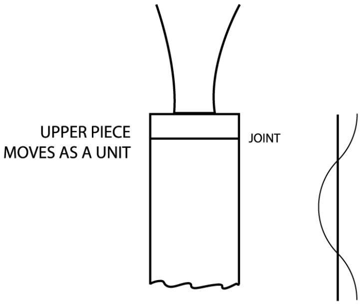


FIGURE 6. WAVELENGTH EFFECT - PURE RESONANCE RESPONSE

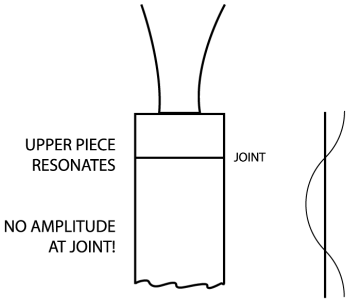
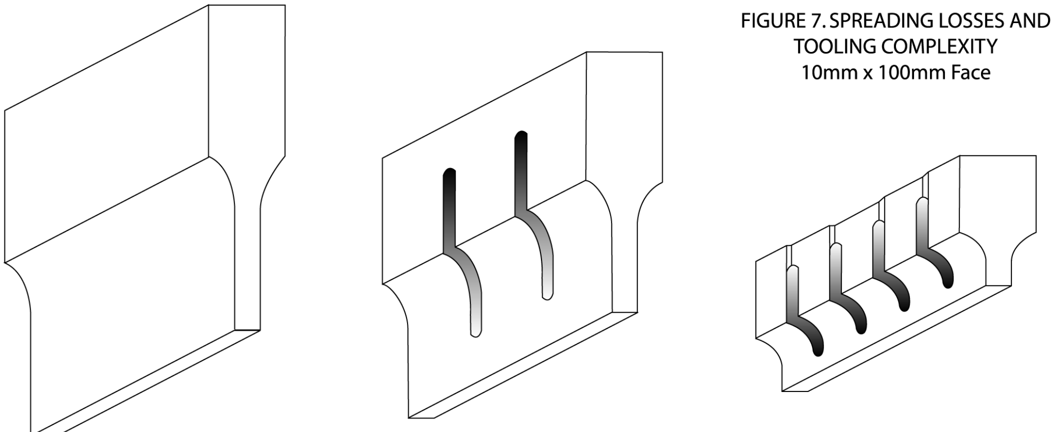


FIGURE 7. SPREADING LOSSES AND TOOLING COMPLEXITY
10mm x 100mm Face



15 KHZ

20 KHZ

40 KHZ