Phase velocity method for guided wave measurements in composite plates.

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1.- Introduction
Objectives

- Evaluation of composite plates using the phase velocity method applied to guided waves. (SV and SH).
  - Determination of elastic constant
  - Flaw evaluation (lamination)
2.- Theoretical model
Preliminary considerations

**SV Waves (Lamb Waves)**
- 2D dimension (plane $x_1, x_2$)
- Orthotropic material (plane $x_1, x_2$)

**SH Waves**
- Quasi-isotropic (plane $x_1, x_3$)
2.-Theoretical model

Anisotropic material

Stiffness Matrix

Voigt notation

\[
\begin{pmatrix}
  C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
  C_{22} & C_{23} & 0 & 0 & 0 & 0 \\
  C_{33} & 0 & 0 & 0 & 0 & 0 \\
  C_{44} & 0 & 0 & 0 & 0 & 0 \\
  C_{55} & 0 & 0 & 0 & 0 & 0 \\
  C_{66} & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]
2.-Theoretical model

SV Considerations

<table>
<thead>
<tr>
<th>Voigt notation</th>
<th>Engineering notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{11}$</td>
<td>$E_1$ and $E_2$ Young modulus</td>
</tr>
<tr>
<td>$C_{22}$</td>
<td>$\mu_{12}$ and $\mu_{21}$ Poisson ratio</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>$G_{12}$ Shear Modulus</td>
</tr>
<tr>
<td>$C_{66}$</td>
<td></td>
</tr>
</tbody>
</table>

$$\epsilon = \frac{E_2}{E_1} = \frac{\mu_{21}}{\mu_{12}} \quad \gamma = \frac{G_{12}}{E_1}$$
2.-Theoretical model

Relation Voigt vs engineering notation (2D)

\[
C_{11} = \frac{E_1}{(1 - \mu_{12}\mu_{21})} \\
C_{12} = \frac{\mu_{12}E_1}{(1 - \mu_{12}\mu_{21})} = \frac{\mu_{21}E_2}{(1 - \mu_{12}\mu_{21})} \\
C_{22} = \frac{E_2}{(1 - \mu_{12}\mu_{21})} \\
C_{66} = G_{12} \\
\epsilon = \frac{E_2}{E_1} = \frac{\mu_{21}}{\mu_{12}} \\
\gamma = \frac{G_{12}}{E_1}
\]
2.-Theoretical model

Plane $x_1 x_3$

Quasi-isotropic

$E_1 = E_3$

$\mu_{13} = \mu_{31}$

$C_{33} = C_{44} = G_{13} = \frac{E_1}{(1 + \mu_{13})}$
2.- Theoretical model

Waves equations in $x_1$ $x_2$ plane

\[
\rho \frac{\partial^2 u_1}{\partial t^2} = C_{11} \frac{\partial^2 u_1}{\partial x_1^2} + C_{12} \frac{\partial^2 u_2}{\partial x_1 \partial x_2} + C_{66} \frac{\partial^2 u_1}{\partial x_2^2} + C_{66} \frac{\partial^2 u_2}{\partial x_1 \partial x_2}
\]

\[
\rho \frac{\partial^2 u_2}{\partial t^2} = C_{12} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{22} \frac{\partial^2 u_2}{\partial x_2^2} + C_{66} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + C_{66} \frac{\partial^2 u_2}{\partial x_1^2}
\]

$\rho$ = density

$u_1$ and $u_2$ displacements
2.-Theoretical model

Dispersion relations of SV (Lamb Waves)

\[ \frac{qh}{2} = \pi \frac{h}{\lambda} \sqrt{\frac{a}{2} + \sqrt{\left(\frac{a}{2}\right)^2 - b}} \]

\[ \frac{sh}{2} = \pi \frac{h}{\lambda} \sqrt{\frac{a}{2} - \sqrt{\left(\frac{a}{2}\right)^2 - b}} \]

\[ a = \frac{C_{12}^2}{C_{22} C_{66}} + \frac{2C_{12}}{C_{22}} - \frac{C_{11}}{C_{66}} + \rho c^2 \left( \frac{1}{C_{66}} + \frac{1}{C_{22}} \right) \]

\[ b = \frac{C_{11}}{C_{22}} - \rho c^2 \left( \frac{C_{11} + C_{66}}{C_{22} C_{66}} \right) + \rho^2 c^4 \left( \frac{1}{C_{66} C_{22}} \right) \]

\[ \text{c= phase velocity} \]

\[ \lambda= \text{wavelength} \]

2.-Theoretical model

Dispersion relations of SV (Lamb Waves)...

\[ A_0 = \frac{q}{k} \left[ -\rho c^2 + C_{11} - \frac{C_{12}(C_{12} + C_{66})}{C_{22}} \right] - C_{66} \left( \frac{q}{k} \right)^3 \]

\[ B_0 = \frac{s}{k} \left[ -\rho c^2 + C_{11} - \frac{C_{12}(C_{12} + C_{66})}{C_{22}} \right] - C_{66} \left( \frac{s}{k} \right)^3 \]

\[ C_0 = -\rho c^2 + C_{11} + C_{12} \left( \frac{q}{k} \right)^2 \]

\[ D_0 = -\rho c^2 + C_{11} + C_{21} \left( \frac{s}{k} \right)^2 \]

\[ k = \frac{2\pi}{\lambda} \]
2.-Theoretical model

\[ C/C_1 = \frac{C_1}{\sqrt{E_1/\rho}} \]

Dispersion curves Case 1

Blue
\[ \varepsilon = 0.5 \quad \gamma = 0.2 \]

Red
\[ \varepsilon = 0.1 \quad \gamma = 0.2 \]

\[ \varepsilon = \frac{E_2}{E_1} = \frac{\mu_{21}}{\mu_{12}} \]

\[ \gamma = \frac{G_{12}}{E_1} \]
2.-Theoretical model

\[ C/C_1 = \sqrt{E_1/\rho} \]

Dispersion curves Case 2

Blue
\[ \epsilon = 0.5 \quad \gamma = 0.2 \]

Red
\[ \epsilon = 0.5 \quad \gamma = 0.02 \]

\[ \epsilon = \frac{E_2}{E_1} = \frac{\mu_{21}}{\mu_{12}} \quad \gamma = \frac{G_{12}}{E_1} \]
2.-Theoretical model

Influence of Transducer ("one side")

3.- Materials and Methods
3.-Materials and Methods

CFRP Laminates

Sample 1

CFRP: -45/+45/90/0/0/90/+45/-45 / “Symmetrical”

660x460x2.9 mm

Sample 2

Cardboard

Quasi-symmetrical

\( \rho = 1.25 \text{ g/cm}^3 \)
3.-Materials and Methods

Flaw embedded ("lamination")
3.-Materials and Methods

Equipment

Ultrasonic System Sitau, Dasel SL
Labview GUI

Burst excitation
Pulse Transmission mode
3.-Materials and Methods

Transducers

SH Transducer
(made in Tecnalia)

SV – L Transducer
Panametrics
Rx= 0.5 MHz (V191)
Tx= 1 MHz (V194)
3.-Materials and Methods

SH transducer. Piezoelectric material

FEM Simulation

Impedance module
3.- Materials and Methods

Transducers

Transducers for flaw detection (longitudinal)

Tx  Rx
3.-Materials and Methods

Basic Configuration
3.-Materials and Methods

Basic Configuration
3.-Materials and Methods

Phase velocity method

In pulses!!!

Slope = 1/phase velocity (C)

Guided waves

4.- Results
4.-Results

$S_0$ Mode 250 KHz

$C_S = 5455 \pm 61 \text{ m/s}$

$h/\lambda = 0.13$ !!!!
4.-Results

Fundamental Poisson ratio

$\mu_{13} = \frac{1}{2} \left( \frac{C_1}{C_T} \right)^2 - 1$

$\mu_{13} = 0.39$

Sample No. 1

$C_T = 3267 \pm 41 \text{ m/s}$

T Mode 200 KHz
4.-Results

A₀ Mode. Evolution with distance

Initial Pulse

Tx

Rx

Sample No. 1

60 KHz

Sample No. 1
4.-Results

Evolution with distance

- for phase velocity
- for group velocity

100 mm

110 mm

120 mm

A₀ mode 60 KHz

Sample No. 1
4.-Results

A0 mode. Some examples

Sample No. 1

C=1202±18 m/s

976±7 m/s

A₀ Mode 150 KHz

A₀ Mode 60 KHz
4.-Results

A₀ Mode. Experimental dispersion curve
Sample No 1
4.-Results

Experimental vs theoretical

$C/C_1$ vs $h/\lambda$

$\varepsilon = 0.2$
$\gamma = 0.07$
$\mu_{12} = 0.4$
4.-Results

Elastic Constants obtained

**Engineering constants**

<table>
<thead>
<tr>
<th>$E_1$ (GPa)</th>
<th>$E_2$ (GPa)</th>
<th>$G_{13}$ (GPa)</th>
<th>$G_{12}$ (GPa)</th>
<th>$u_{13}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.2</td>
<td>7.4</td>
<td>13.3</td>
<td>2.6</td>
<td>0.39</td>
</tr>
</tbody>
</table>

**Voigt**

<table>
<thead>
<tr>
<th>$C_{11}$ (GPa)</th>
<th>$C_{22}$ (GPa)</th>
<th>$C_{12}$ (GPa)</th>
<th>$C_{66}$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.4</td>
<td>7.7</td>
<td>3</td>
<td>2.6</td>
</tr>
</tbody>
</table>
4.-Results

Flaw evaluation

Sample No. 2
4.-Results

Flaw evaluation

Rx1

Rx2

Rx1

Rx2

New cursor position

Sample No. 2

Tx

Rx1

Rx2

Sample No. 2
4.-Results

Numerical evaluation

If $c=1.1 \text{ mm/\mu seg at 100 KHz}$

Then

A = Time difference in flawless position = 18.125 \mu seg

B = Time difference in flaw position = 24.875 \mu seg

Velocity over the flaw = \((A/B) \times 1.1 = 0.801 \text{ mm/\mu seg} = 801 \text{ m/s}\)
4.- Results

Numerical evaluation vs. theoretical model

C/C_1

h/λ

h=1mm in the flaw region
Conclusions

• The phase velocity method was used for determination of elastic constants and the evaluation of plates.
• It was possible to use this method with SH and SV guided waves.
• A flaw evaluation (lamination) could be possible with this method.
• But the elastic model should be changed. (viscoelastic model)
Some extra details

• The “elastic” constants should be evaluated as viscoelastic “constants”

\[ C_{ij} = C_{ijR} + iC_{ijI} \]

\[ C_{ij}(\omega) = C_{ijR}(\omega) + iC_{ijI}(\omega) \]

Dispersion
  • Geometrical dispersion
  • Viscoelastic dispersion
Some extra details

Viscoelastic dispersion model

Thank you